EEC 4230 - Mobile Communication Systems

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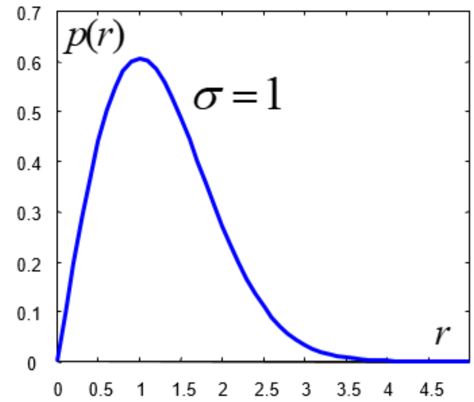
Lecture 7: Mobile propagation: Small-scale multipath fading-II

Outline

Multipath Fading Distributions

(1) Rayleigh Fading Distributions

- Baseband received signal can be written as $y(t) = y_I(t) + jy_Q(t)$.
- When LOS is absent, the I-Q (In-Phase and Quadrature) components of the received signal follow zero-mean Gaussian distribution. Thus the **envelope**, $r(t) = |y(t)|$, of the received signal can be modeled as a zero-mean Rayleigh random variable with probability density function (pdf):
  $p(r) = \frac{r}{\sigma^2} e^{-\frac{r^2}{2\sigma^2}}, \qquad r \geq 0$
- The cumulative distribution function (CDF), is the probability that the received signal envelope does not exceed a value R:
  $p(r \leq R) = \int_0^R p(r)dr = 1 - e^{-\frac{R^2}{2\sigma^2}}$

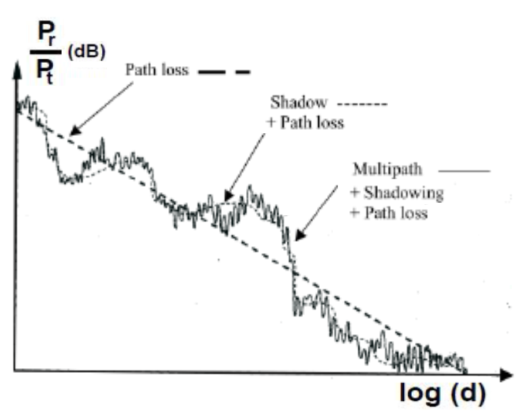


σ: the rms value of the received voltage signal
σ^2: the time-average power of the received signal.

Multipath Fading Distributions

(1) Rayleigh Fading Distributions

- Baseband received signal **power** is given by $P_r(t) = r^2(t)$
- Therefore, The received signal power for a Rayleigh fading model, is exponentially distributed with mean $E[P_r(t)] = 2\sigma^2$.
- Probability density function (PDF) of the received power $x = r^2$ is given by:
  $p(x) = \frac{1}{2\sigma^2} e^{-\frac{x}{2\sigma^2}}, \qquad x \geq 0$
- Note that $E[P_r(t)]$ is the received signal power based on path loss and shadowing alone, while the pdf above models the random variations around $E[P_r(t)]$, based on small-scale fading (see Figure).



Example 1: Homework

Given that the received signal amplitude r, over a wireless channel, is a Rayleigh distributed random variable.

1. Compute the mean value, and the variance of r.
2. Calculate also the median value, what does this signify?.

Solution

1. $r_{mean} = E[r] = \int_0^\infty rp(r)dr = \sigma\sqrt{\frac{\pi}{2}}$.
   $var(r) = E[(r - r_{mean})^2] = E[r^2 - 2rr_{mean} - r^2_{mean}] = \int_0^\infty r^2 p(r)dr - \sigma^2 \frac{\pi}{2} = \sigma^2(2 - \frac{\pi}{2}) = 0.4292\sigma^2$.
2. Median value represents the value of r for which the CDF = 0.5.
   Solve CDF $= \int_0^{r_{median}} p(r)dr = 0.5 \Rightarrow r_{median} = 1.77\sigma$

Example 2:

Consider a channel with Rayleigh fading envelope and average received power $E[P_r(t)] = 20$ dBm. Find the probability that the received power is below 10dBm.

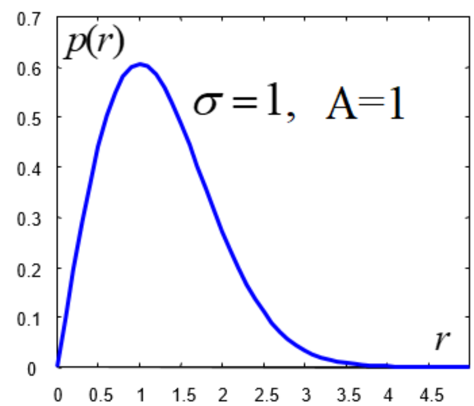
Solution

1. Note that $E[P_r(t)] = 20$dBm, i.e. $= 100$ mW. We want to find the probability that $r^2 < 10$ dBm i.e, < 10 mW.
   $p(r^2 < 10) = \int_0^{10} \frac{1}{100} e^{-\frac{x}{100}} dx = 0.095$

Multipath Fading Distributions

(2) Ricean Fading Distributions

- Baseband received signal can be written as $y(t) = y_I(t) + jy_Q(t)$.
- When LOS is present, the I-Q components of the received signal follow Gaussian distribution with mean $E[y(t)] = A$.
- Thus the envelope of the received signal can be modeled as a Ricean random variable with $E[y(t)] = A$ and probability density function
  $p(r) = \frac{r}{\sigma^2} e^{-\frac{r^2+A^2}{2\sigma^2}} I_0(\frac{Ar}{\sigma^2}), \qquad r \geq 0$
- where A = amplitude of the LOS signal, I_0 = Bessel function of the first kind, of order 0.
- It becomes a Rayleigh fading model if $A = 0$ (i.e., no LOS).



Rice factor: $K = 10\log_{10}\left(\frac{A^2}{2\sigma^2}\right)$ dB

Multipath Fading Distributions

(2) Ricean Fading Distributions

- The average received power in Ricean fading distribution is given by $\bar{P}_r = E[r^2] = \int_0^\infty r^2 p(r)dr = A^2 + 2\sigma^2$
- Where $2\sigma^2$ is the average power in the non-LOS multipath components, and A^2 is the power in the LOS component.
- Thus the Rice factor K is the ratio of power in the LOS to the power in the non-LOS components.
- Note: $A^2 = K \times 2\sigma^2 = K \times 2\sigma^2 \times \frac{A^2+2\sigma^2}{A^2+2\sigma^2} = K \times \frac{A^2+2\sigma^2}{\frac{A^2}{2\sigma^2}+1} = K \times \frac{\bar{P}_r}{K+1}$
  and $2\sigma^2 = \frac{\bar{P}_r}{K+1}$
- Using these relations, the Ricean pdf can be written as:
  $p(r) = \frac{2r(K+1)}{\bar{P}_r} e^{-K-\frac{(K+1)r^2}{\bar{P}_r}} I_0\left(2r\sqrt{\frac{K(K+1)}{\bar{P}_r}}\right), \qquad r \geq 0$
- Examples of Rician Fading channel:
  (1) Microcellular channel (2) Satellite channel (3) Indoor channel

Multipath Fading Distributions

(3) Nakagami Fading Distributions

- Gives more generalized fading statistics than Rayleigh and Ricean models.
- Practical measurements show that Nakagami distribution models signal received over mobile channels, more accurately.
- The Nakagami pdf is given by
  $p(r) = \frac{2}{\Gamma(m)}\left(\frac{m}{\bar{P}_r}\right)^m r^{2m-1} e^{-\frac{mr^2}{\bar{P}_r}}, \qquad m \geq 0.5, r \geq 0$
- where $\Gamma(m) = \int_0^\infty y^{m-1}e^{-y}dy$ is the gamma function, and $\bar{P}_r = E[r^2]$
- m is called the Nakagami fading parameter (usually $m \geq 0.5$)
- $m = 1$, Nakagami fading becomes Rayleigh fading model.
- $m \geq 2$, Less severe fading can be modeled.
- As $m \to \infty$, non-fading or AWGN channel, can be modeled.
- For $m = \frac{(K+1)^2}{(2K+1)}$, Ricean fading with factor K can be modeled.

Multipath Fading Distributions

(3) Nakagami Fading Distributions

- Thus the Nakagami distribution can model Rayleigh, Ricean, AWGN (no fading), and more general fading in more or less severer than Rayleigh and Ricean.
- Therefore, Measurements in different environments can fit the Nakagami distribution by choosing appropriate m.
- The power distribution for Nakagami fading, obtained by making change of variable $x = r^2$ in above pdf, is given by

$$p(x) = \left(\frac{m}{\bar{P}_r}\right)^m \frac{x^{m-1}}{\Gamma(m)} e^{-\frac{mx}{\bar{P}_r}}, \qquad m \geq 0.5, x \geq 0$$

Level Crossing and Average Fade durations

There are two important fading parameters:

Level Crossing Rate (LCR) and Average Fade durations (AFD)

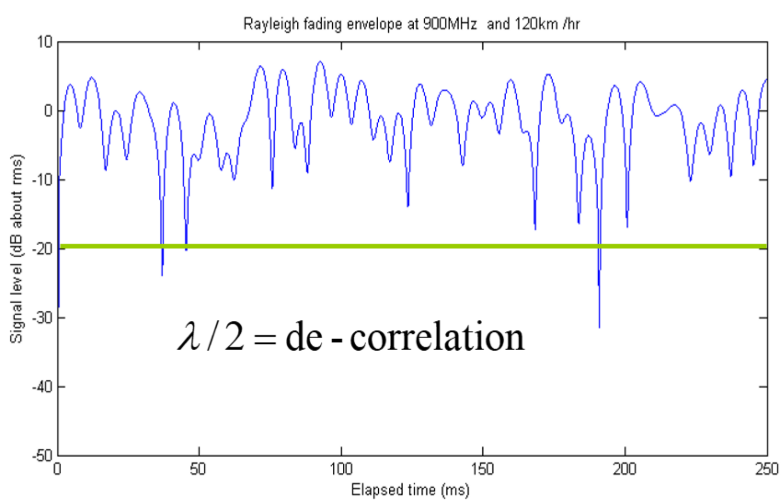
1. Level crossing rate (LCR): is the average number of level crossings or fades(or the expected rate at which the Rayleigh fading envelope crosses a specified level R), when envelope is in a positive going direction. (i.e. $r_1 - r_2$ = +ve).
2. Average Fade Duration (AFD): The duration of these fade (or it is the average period of time for which the received signal is below a specified level R).

The number of level crossing per second is given by

$$N_R = \int_0^\infty r' p(R, r') dr' = \sqrt{2\pi} f_m \rho e^{-\rho^2}$$

Where r' is the slope of r or the time derivative of r, and $p(R, r')$ is the joint pdf of r and r' at $r = R$. Also $\rho = \frac{R}{R_{rms}}$, where R_{rms} is the local rms amplitude of the fading envelope. Note also: $\rho = \sqrt{\frac{P_0}{P_r}}$ where P_0 is a target received power (receiver sensitivity).

Level Crossing and Average Fade durations



Rayleigh fading simulation, $fc = 900$ MHz, and Mobile Speed = 120 km/hr

Exercise: at threshold level -20dB about RMS, estimate N_R for this simulation.

Example 3:

For a Rayleigh fading signal,

1. Compute the positive-going level crossing rate for $\rho = 1$, when the maximum Doppler frequency, f_m is 20Hz.
2. What is the maximum velocity of the mobile for this Doppler frequency if the carrier frequency is 900 MHz?.

Solution

1. $N_R = \sqrt{2\pi}(20)(1)e^{-1^2} = 8.44$ crossing per seconds.
2. The maximum velocity of the mobile can be obtained using the Doppler relation, $f_m = \frac{v}{\lambda}$
   Thus $v = 20$ Hz $\times \frac{1}{3}$ m =6.66 m/s = 24 km/hr

Level Crossing and Average Fade durations

Average Fade durations (AFD)

1. For a Rayleigh fading signal, the average fade duration is given by

$$\bar{\tau}_{AFD} = \frac{1}{N_R} p(r \leq R) = \frac{e^{\rho^2} - 1}{\rho f_m \sqrt{2\pi}}$$

2. The average fade duration helps to determine the likely number of signaling bits that may be lost when fading occurs.

Example 4:

1. Find the average fade duration for a threshold level of $\rho = 0.707$ when the Doppler frequency is 20Hz. For a binary digital modulation with data rate of 50 bps, is this Rayleigh fading scenario slow or fast?.
2. What is the average number of bit errors per second for the given data rate for the case $\rho = 0.1$ [assuming that a bit error occurs whenever any portion of a bit encounters fading]?.

Solution

1. $\bar{\tau}_{AFD} = \frac{e^{0.707^2}-1}{0.707\times 20\times\sqrt{2\pi}} = 18.3$ ms
   For $R_b = 50$ bps, $T_b = 20$ ms, which is $> \bar{\tau}_{AFD}$. Thus the signal undergoes fast Rayleigh fading.
2. For $\rho = 0.1$, we have $\bar{\tau}_{AFD} = 0.002$ s = 2 ms. This is less than duration of one bit. Therefore, only one bit on average will be lost when a fading event occur.
3. For $\rho = 0.1$, $N_R = 4.96$ crossings per second. Thus there are total of 5 bits in error per sec., resulting in BER=(5/50)=0.1

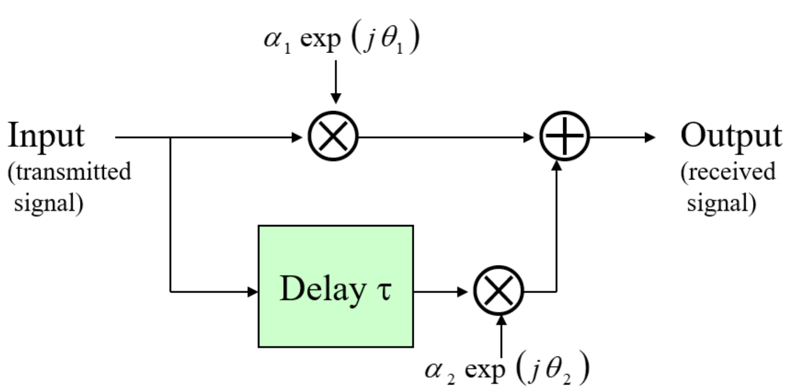
Frequency-Flat and Freq-selective Fading Models

(1) Two-ray frequency-selective fading model

It models the effect of multipath delay spread as well as fading.

1. The impulse response of the channel model is given as:

$$h(t,\tau) = \alpha_1 e^{j\theta_1}\delta(t) + \alpha_2 e^{j\theta_2}\delta(t-\tau)$$



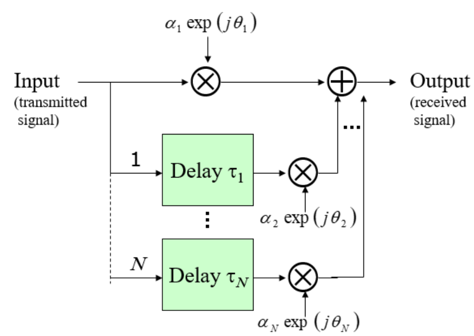
Frequency-Flat and Freq-selective Fading Models

(1) Two-ray frequency-selective fading model

1. α_1 and α_2 are independent and Rayleigh distributed.
2. θ_1 and θ_2 are independent and uniformly distributed over $[0,2\pi]$.
3. τ is the time delay between the two rays.
4. Setting $\alpha_2 = 0$, a flat Rayleigh fading channel is obtained.
5. By varying τ, we can create a wide range of frequency selective fading effects.

(2) N-ray Rayleigh fading model

Develop an $(N+1)$ -ray mobile channel simulator as shown below:



Frequency-Flat and Freq-selective Fading Models

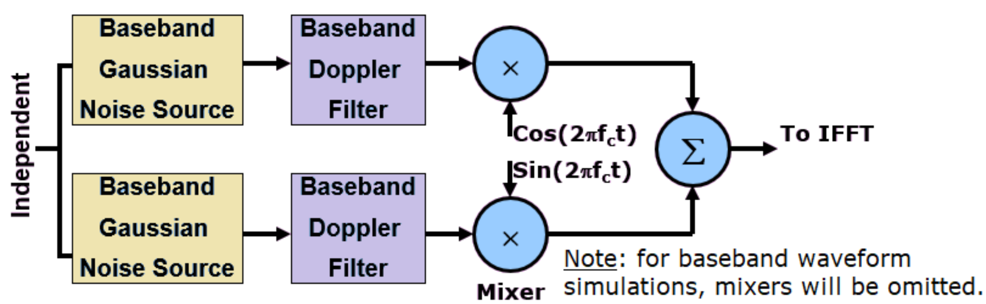
(3) Clarke's model for flat fading (freq domain)

1. While statistical models (Rayleigh, Ricean, etc.) directly predict received signal envelope in TD, Clarke's model predicts the PSD of the received signal first in FD then it is used to produce a TD waveform. It is widely used to model narrow-band channel in IS-54 (US digital cellular system).
2. Given that the source, S, transmits a continuous wave signal of frequency f_c. Then the instantaneous frequency of the received signal component arriving at an angle θ at the mobile is: $f = f_c + \frac{v}{\lambda}\cos\theta$. Thus $\theta = \cos^{-1}\left(\frac{f-f_c}{f_{D_{max}}}\right)$
3. Clarke's model uses statistical characteristic of electromagnetic waves to show that the received power spectral density (PSD), $S(f)$, is proportional to $\frac{\partial\theta}{\partial f}$.
4. So, $S(f) \propto \frac{\partial\theta}{\partial f} = \frac{\partial}{\partial f}\cos^{-1}\left(\frac{f-f_c}{f_{D_{max}}}\right) = \frac{1}{\sqrt{1-\left(\frac{f-f_c}{f_{D_{max}}}\right)^2}}$ i.e. $S(f) = \frac{K}{\sqrt{1-\left(\frac{f-f_c}{f_{D_{max}}}\right)^2}}$
5. In the text book, pages 214-218, show that $K = \frac{1.5}{\pi f_{D_{max}}}$

Frequency-Flat and Freq-selective Fading Models

(3) Simulation of Clarke's fading model using Doppler filter

1. Baseband received signal: $y(t) = y_I(t) + jy_Q(t)$, and $r(t) = |y(t)|$.
2. Generate two independent, random complex Gaussian source.
3. Then use spectral filter defined by $S(f)$ in the previous slide, to shape the random signals in frequency domain.
4. Time domain waveform of the resulting fading generated can be produced by using an inverse fast Fourier transform (IFFT) at the last stage of the simulator.

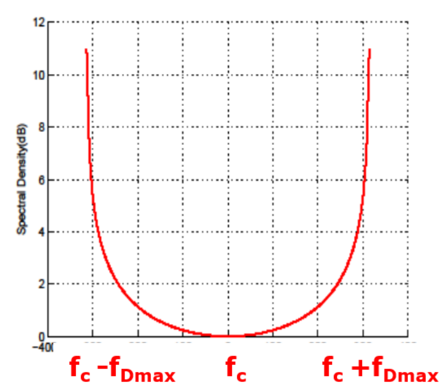


Frequency-Flat and Freq-selective Fading Models

Clarke's model-flat fading (FD)-Computer Implementation

1. Specify number of frequency points N to represent $\sqrt{S(f)}$, and the $f_{D_{max}}$.
2. Calculate frequency spacing between adjacent spectral lines as $\Delta f = \frac{2f_{D_{max}}}{(N-1)}$. Time duration of the fading waveform is $T = \frac{1}{\Delta f}$.
3. Generate complex Gaussian random variables for each of the $N/2$ positive frequency component of the noise source.
4. Construct the negative frequency components of the noise source by conjugating positive frequency values & assigning these at the negative frequency values.
5. Multiply the I and Q noise sources by fading spectrum $\sqrt{S(f)}$. (Freq domain operation ends here.)

Usually N is power of 2, i.e. $N = 2^k$, where $k = 1, 2, 3$.



Doppler power Spectrum

[1] J. Smith, "A Computer Generated Multipath Fading Simulation for Mobile Radio," IEEE TVT, 1975.

Frequency-Flat and Freq-selective Fading Models

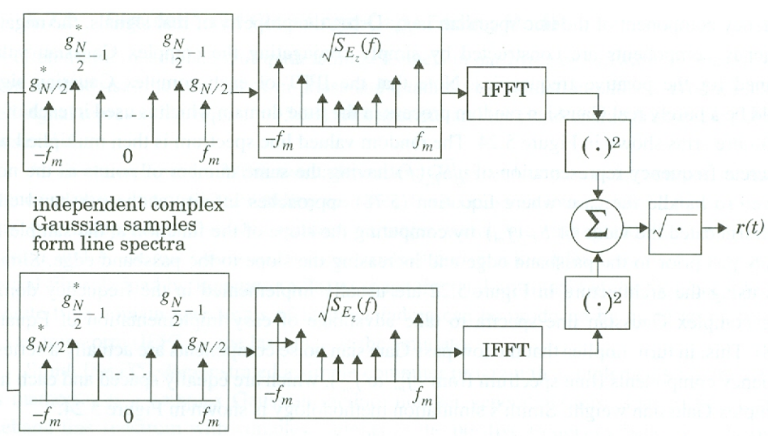
(3) Clarke's model-flat fading (FD)-Computer Implementation

1. Perform an IFFT on the resulting frequency domain signals from the I-Q parts to get two N-length time series, & add the squares of each signal point in time to create an N-point time series.
2. Take square root of the sum obtained in 6 to obtain a simulated Rayleigh fading signal, to model the expression: $r(t) = |y(t)| = \sqrt{(y_I^2(t) + y_Q^2(t))}$, with proper Doppler spread and time correlation.
3. By making a frequency component dominant in amplitude within $\sqrt{S(f)}$, and at $f = 0$, the fading is changed from Rayleigh to Ricean.

Example 5:

1. Generate Fig 5.15 in [Ref 1] – typical Rayleigh fading envelope at 900MHz, and $v = 120$ km/hr.

Frequency-Flat and Freq-selective Fading Models

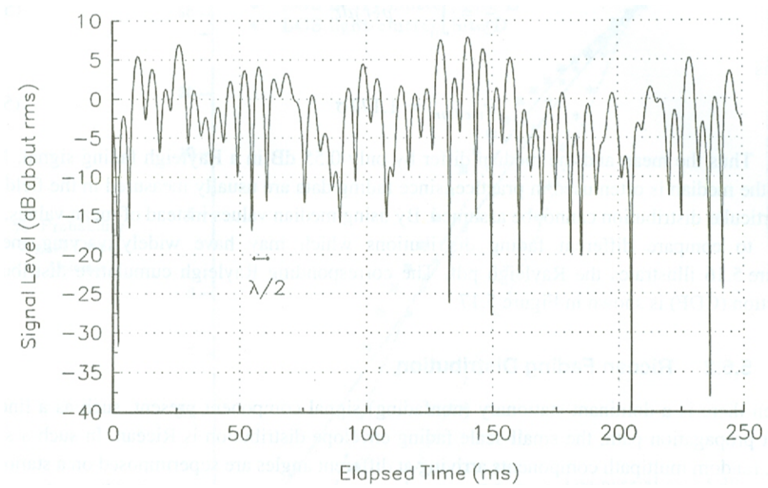


Frequency domain of a Rayleigh fading simulator at baseband

[2] T.S. Rappaport, "Wireless Communications: principles and practice," 2Ed., Prentice Hall, NJ, USA.

[3] V. Fun, T.S. Rappaport, B. Thoma, "Bit Error Simulation of $\pi/4$-DQPSK Mobile Radio Communication using Two-ray and Measurement-based Impulse Response Models," IEEE JSAC, 1993.

Frequency-Flat and Freq-selective Fading Models



Typical Rayleigh Fading envelope at 900 MHz, and $v = 120$ km/hr

[2] T.S. Rappaport, "Wireless Communications: principles and practice," 2Ed., Prentice Hall, NJ, USA.

[3] V. Fun, T.S. Rappaport, B. Thoma, "Bit Error Simulation of $\pi/4$-DQPSK Mobile Radio Communication using Two-ray and Measurement-based Impulse Response Models," IEEE JSAC, 1993.

Frequency-Flat and Freq-selective Fading Models

Clarke's model for flat fading (time domain)

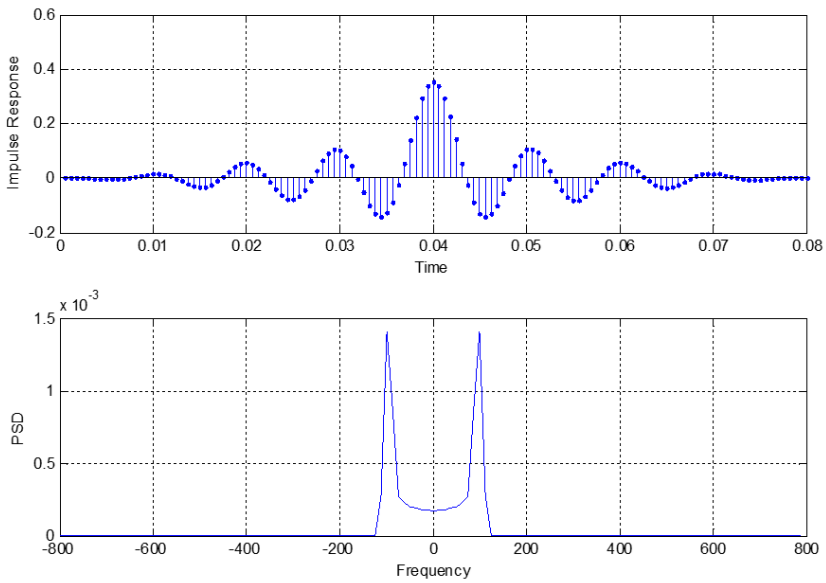
1. It defines the complex channel gain, for NLOS, frequency flat, and 2-D isotropic scattering assumptions as [3]
   $h(t) = \sqrt{\frac{2}{N}} \sum_{n=1}^{N} e^{j(2\pi f_{D_{max}} t \cos\alpha_n + \phi_n)}$
2. N is the number of multipaths, $\phi_n \sim$ uniform $(-\pi, \pi)$, and $\alpha_n \sim$ uniform $(-\pi, \pi)$ are the phase and amplitudes of the n^{th} multipath component.

Jakes' Fading model

1. the high degree of randomness in equation above is not desirable for efficient simulation. Thus, Jakes proposed the following sum of sinusoid model to simulate $h(t)$, & is widely used.
   $h_I(t) = \sqrt{2}\cos(2\pi f_{D_{max}} t) \times 2\sum_{n=1}^{M} \cos(\frac{2\pi n}{M}) \cos\left(2\pi f_{D_{max}} \cos(\frac{2\pi n}{4M+2})t\right)$
   $h_Q(t) = 2\sum_{n=1}^{M} \sin(\frac{2\pi n}{M}) \cos\left(2\pi f_{D_{max}} \cos(\frac{2\pi n}{4M+2})t\right)$

Other recent models: Zheng et al. [8], Young and Beaulieu's method for computational efficient Rayleigh fading sim.[Ref7].

Frequency-Flat and Freq-selective Fading Models



Jake's Fading Model [4]

[4] W. C. Jakes, "Microwave Mobile Communications," Wiley-IEEE Press, May 1994.